Sankalp IIT

A MUST DO TEST SERIES FOR SURE SHOT SUCCESS IN JEE MAIN AND ADVANCED

Solutions All India Test Series
Test-2

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(4)	31.	(3)	61.	(1)
2.	(4)	32.	(4)	62.	(2)
3.	(2)	33.	(1)	63.	(1)
4.	(1)	34.	(4)	64.	(1)
5.	(3)	35.	(3)	65.	(2)
6.	(1)	36.	(3)	66.	(3)
7.	(1)	37.	(4)	67.	(1)
8.	(2)	38.	(3)	68.	(2)
9.	(4)	39.	(4)	69.	(2)
10.	(4)	40.	(3)	70.	(4)
11.	(4)	41.	(4)	71.	(2)
12.	(3)	42.	(1)	72.	(1)
13.	(1)	43.	(3)	73.	(3)
14.	(4)	44.	(4)	74.	(1)
15.	(3)	45.	(1)	75.	(1)
16.	(3)	46.	(3)	76.	(1)
17.	(2)	47.	(2)	77.	(4)
18.	(4)	48.	(3)	78.	(4)
19.	(2)	49.	(2)	79.	(3)
20.	(3)	50.	(1)	80.	(3)
21.	(4)	51.	(2)	81.	(4)
22.	(1)	52.	(2)	82.	(2)
23.	(3)	53.	(3)	83.	(4)
24.	(3)	54.	(3)	84.	(3)
25.	(2)	55.	(3)	85.	(1)
26.	(2)	56.	(1)	86.	(2)
27.	(2)	57.	(1)	87.	(1)
28.	(2)	58.	(3)	88.	(1)
29.	(2)	59.	(2)	89.	(3)
30.	(2)	60.	(3)	90.	(3)

Prepared By:
Er. Vineet Loomba (IIT Roorkee)

PHYSICS

1. Answer (4)
2. Answer (4)
3. Answer (2)
4. Answer (1)
5. Answer (3)
6. Answer (1)
7. Answer (1)
8. Answer (2)
9. Answer (4)
10. Answer (4)
11. Answer (4)
12. Answer (3)
13. Answer (1)
14. Answer (4)
15. Answer (3)

    While weight ρvg of liquid is overflowed and a reaction of buoyant force equal to ρvg acts on the liquid.

16. Answer (3)

$$\rho gh = 2T\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\Rightarrow h = \frac{2T\left(r_2-r_1\right)}{\rho g r_1 r_2}$$

17. Answer (2)
18. Answer (4)
19. Answer (2)
20. Answer (3)
21. Answer (4)
22. Answer (1)
23. Answer (3)
24. Answer (3)

Solution from Q.22 to Q.24

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\left(\frac{27}{8}\right)^{\frac{2}{3}} = \frac{r_2}{r_1} \Rightarrow r_2 = 10^5\left(\frac{3}{2}\right)^2$$

$$r_2 = 10^5\left(\frac{9}{4}\right) = 2.25\times10^5 \text{ km}$$

$$v_2 = \frac{2\pi\times9\times10^5}{4\times27}$$

$$= \frac{\pi}{6}\times10^5 \text{ km/h}$$

Angular speed of S_2 as observed by astronaut in S_1 is

$$\omega_r = \frac{|v_2-v_1|}{|r_2-r_1|}$$

$$v_1 = \frac{2\pi\times10^5}{8} = \frac{\pi}{4}\times10^5$$

$$\omega_r = \frac{\left|\frac{1}{6}-\frac{1}{4}\right|\pi\times10^5}{\left(\frac{9}{4}\times10^5-10^5\right)} = \frac{\left(\frac{1}{12}\right)\pi}{\left(\frac{5}{4}\right)} = \frac{\pi}{15} \text{ rad/h}$$

25. Answer (2)

The centre of mass will follow the same path

$$90 = \frac{1\times0+2\times45+3\times x}{6}$$

$90 \times 6 = 90 + 3x$

$90 \times 5 = 3x \Rightarrow x = 150$ m

26. Answer (2)

$$m_0 = AH\rho \Rightarrow H = \frac{m_0}{A\rho}$$

when 50% of liquid has drained out

In this position $h = \frac{H}{2}$

$$\therefore\ v = \sqrt{2g\frac{m_0}{2A\rho}} = \sqrt{\frac{m_0 g}{A\rho}}$$

Force is exerted on the container

Force = ρ × (area of hole) × v^2

$$\frac{m_0}{2}\times a = \frac{\rho A}{400}\times\frac{m_0 g}{A\rho}$$

$$a = \frac{g}{200}$$

27. Answer (2)

The liquid rise in capillary tube is $h_0 = \frac{2T}{\rho gr}$

$$\Delta U = \frac{mgh_0}{2} = \left(\rho g\pi r^2 h_0\right)\left(\frac{h_0}{2}\right) = \frac{2\pi T^2}{\rho g}$$

28. Answer (2)

$$\frac{K_R}{K_T}=\frac{2}{5}$$

Energy at bottom

$$E_B = \frac{7}{5}K_T = \frac{7}{5}\left(\frac{1}{2}mv^2\right) = \frac{7}{5}\times\frac{1}{2}\times\left(\sqrt{\frac{200}{7}}\right)^2 = 20\text{ J}$$

Kinetic energy at top $K_C = E_B - mgh = 10$ J

$$\frac{K_R}{K_T}=\frac{2}{5}$$

$$\frac{K_T}{K_R}=\frac{5}{2}$$

$$\frac{10}{K_R}=\frac{7}{2}$$

$$\Rightarrow K_R = \frac{20}{7}$$

29. Answer (2)

Total kinetic energy = $\frac{5}{6}mv^2$

Kinetic energy of bottom half = $\frac{mv^2}{6}$

$\Rightarrow$ Kinetic energy of upper half = $\frac{2}{3}mv^2$

30. Answer (2)

Work done by friction force = $+\mu mgx$

Work done by gravitational force = Mgy

$$\frac{1}{2}Mv^2 = Mgx - \mu Mgy$$

$$v = \sqrt{2(gx - \mu gy)}$$

CHEMISTRY

31. Answer (3)

Use $\Delta G^o = \Delta H^o - T\cdot\Delta S^o$

& $\Delta G^o = -RT\ln k_p$

32. Answer (4)

$$\underset{1-\alpha}{PCl_5} \rightleftharpoons \underset{\alpha}{PCl_3} + \underset{\alpha}{Cl_2}$$

$$K_p = \frac{(\alpha)^2}{(1+\alpha)^2}\frac{P^2(1+\alpha)}{(1-\alpha)P}$$

$$K_p = \frac{0.5^2\cdot 4}{0.75} = \frac{0.25\times 4}{0.75} = \frac{4}{3}$$

$$\frac{4}{3} = \frac{0.64\times P}{0.36}$$

$$P = \frac{4}{3}\times\frac{9}{16}$$

P = 0.75 atm

33. Answer (1)
34. Answer (4)
35. Answer (3)
36. Answer (3)
37. Answer (4)

Metal is an alkaline earth metal.

38. Answer (3)
39. Answer (4)
40. Answer (3)
41. Answer (4)
42. Answer (1)
43. Answer (3)
44. Answer (4)
45. Answer (1)
46. Answer (3)
47. Answer (2)
48. Answer (3)
49. Answer (2)
50. Answer (1)
51. Answer (2)
52. Answer (2)
53. Answer (3)
54. Answer (3)
55. Answer (3)
56. Answer (1)
57. Answer (1)
58. Answer (3)
59. Answer (2)
60. Answer (3)

MATHEMATICS

61. Answer (1)

$x^3 + 2x^2 + 2x + 1 = 0$

$x^3 + x^2 + x^2 + x + x + 1 = 0$

$x^2(x + 1) + x(x + 1) + 1(x + 1) = 0$

$(x + 1)(x^2 + x + 1) = 0$

$\Rightarrow \quad x = -1$ or ω or ω^2

$x = -1$, does not satisfy $1 + x^{130} + x^{1998} = 0$

$x = \omega$, $1 + \omega^{130} + \omega^{1998} = 1 + \omega + \omega^0 \neq 0$

ω, ω^2 do not satisfy this equation, so number of common root is 0.

62. Answer (2)

We have,

$|z^2 - 4| \geq |z^2| - |4|$, $|z| > 2$

$\Rightarrow \quad 2|z| \geq |z|^2 - 4$

or $\quad |z|^2 - 2|z| - 4 \leq 0$

or $\quad |z^2| - 2|z| + 1 \leq 5$

or $\quad (|z| - 1)^2 \leq 5$

$\Rightarrow \quad -\sqrt{5} \leq |z| - 1 \leq \sqrt{5}$

$\Rightarrow \quad 1 - \sqrt{5} \leq |z| \leq 1 + \sqrt{5}$

So, the maximum value of $|z|$ will be equal to $\left(1+\sqrt{5}\right)$

63. Answer (1)

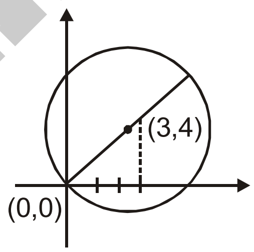
$$\frac{z_1}{z_2} = ki$$

$$\Rightarrow \frac{2z_1}{3z_2} = \frac{2ki}{3}$$

$$\Rightarrow \frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{2ki + 3}{2ki - 3}$$

$$\left|\frac{2z_1 + 3z_2}{2z_1 - 3z_2}\right| = \left|\frac{2ki + 3}{2ki - 3}\right| = \frac{\sqrt{4k^2 + 9}}{\sqrt{4k^2 + 9}} = 1$$

64. Answer (1)



z lies on the boundary of the circle having centre at (3, 4) and radius equal to five. $|z - i|$ represents the distance between z and i. The distance between (3, 4) and (0, 1) is equal to $\sqrt{(3-0)^2 + (4-1)^2} = 3\sqrt{2}$

So the maximum value of $|z - i|$ will be equal to $\left(3\sqrt{2} + 5\right)$ units.

65. Answer (2)

The first digit should be either one or greater than one. Required number of 6 digit numbers is ${}^9C_6$ = $\frac{9!}{6!3!} = \frac{9 \times 8 \times 7}{6} = 84$

66. Answer (3)

$$\sum_{J=0}^{100} {}^{100}C_j (x-3)^{100-j} \cdot 2^j$$

$= \quad {}^{100}C_0 (x-3)^{100} 2^0 + {}^{100}C_1 (x-3)^{99} 2^1 + ...$

$.....+ {}^{100}C_{100} (x-3)^0 . 2^{100}$

$= \quad ((x - 3) + 2)^{100}$

$= \quad (x - 1)^{100}$

General term will be

$T_{r+1} = {}^{100}C_r (x)^{100-r} (-1)^r$

Put $100 - r = 53$

$\Rightarrow r = 47$

So, $T_{48} = {}^{100}C_{47} . x^{53} (-1)^{53} = -{}^{100}C_{47} . x^{53} = -{}^{100}C_{53} x^{53}$

67. Answer (1)

$(x + a)^4 (x + b)^5 (x + c)^6$

$= ({}^4C_0 x^4 + {}^4C_1 x^3 a + {}^4C_2 x^2 a^2 + {}^4C_3 x a^3 + {}^4C_4 a^4)$

$({}^5C_0 x^5 + {}^5C_1 x^4 b + {}^5C_2 x^3 b^2 + {}^5C_3 x^2 b^3 + {}^5C_4 x b^4 + {}^5C_5 b^5)$

$({}^6C_0 x^6 + {}^6C_1 x^5 c + {}^6C_2 x^4 c^2 + {}^6C_3 x^3 c^3 + {}^6C_5 x^2 c^4 +$

${}^6C_5 x c^5 + {}^6C_6 c^6)$

The required coefficient of x^{14}

$${}^4C_0 . {}^5C_0 . {}^6C_1 (c) + {}^5C_0 . {}^6C_0 . {}^4C_1 (a) + {}^4C_0 . {}^5C_1 . {}^6C_0 (b)$$

$= \quad 6c + 4a + 5b = 4a + 5b + 6c$

$\Rightarrow \quad k_1 = 4, k_2 = 5, k_3 = 6$

$\Rightarrow \quad k_1 . k_2 . k_3 = 120$

68. Answer (2)

We have,

$$T_{r+1} = {}^5C_r (x)^{5-r} (x^{\log_{10} x})^r$$

$\Rightarrow\ T_{r+1} = {}^5C_r(x)^{5-r}.x^{r\log_{10}x}$

$\Rightarrow\ T_3 = {}^5C_2(x)^3.x^{(2\log_{10}x)}$

$\Rightarrow\ 10^6 = 10.x^3\ \ x^{2\log_{10}x}$

$\Rightarrow\ 10^5 = x^3.\ x^{2\log_{10}x}$

$\Rightarrow\ 10^5 = (x)^{3+2\log_{10}x}$

$\Rightarrow\ 5 = (3 + 2\log_{10}x)\log_{10}x$ (let $\log_{10}x = z$)

(x = 10 using hit & trial method)

$\Rightarrow\ 5 = (3 + 2z)\, z$

$\Rightarrow\ 2z^2 + 3z - 5 = 0,\ 2z^2 + 5z - 2z - 5 = 0,$
$z(2z + 5) - 1(2z + 5), = 0,\ (2z + 5)(z - 1) = 0$

$\Rightarrow\ z = -5/2,\ z = 1$

So $x = 10,\ 10^{-5/2}$

69. Answer (2)

We have,

$(x - a)(x - 10) = 1$

For integral root

$x - a = 1,\ x - 10 = 1$

or $x - a = -1,\ x - 10 = -1$

When, $x - 10 = 1 \Rightarrow x = 11$

$\Rightarrow\ 11 - a = 1 \Rightarrow a = 10$

When, $x - 10 = -1 \Rightarrow x = 9$

$\Rightarrow\ 9 - a = -1$

$\Rightarrow\ a = 10$

In either case, a = 10

70. Answer (4)

Let its imaginary roots be $\alpha \pm i\beta$

$$\frac{1}{(\alpha+i\beta)-1}+\frac{2}{(\alpha+i\beta)-2}+\frac{3}{(\alpha+i\beta)-3}=1 \quad \text{.... (1)}$$

$$\frac{1}{(\alpha-i\beta)-1}+\frac{2}{(\alpha-i\beta)-2}+\frac{3}{(\alpha-i\beta)-3}=1 \quad \text{.... (2)}$$

Subtracting equaiton (1) by equation (2)

$$\left\{\frac{1}{(\alpha+i\beta)-1}-\frac{1}{(\alpha-i\beta)-1}\right\}+2\left\{\frac{1}{\alpha+i\beta-2}-\frac{1}{\alpha-i\beta-2}\right\}+$$

$$3\left\{\frac{1}{\alpha+i\beta-3}-\frac{1}{\alpha-i\beta-2}\right\} = 0$$

$$\Rightarrow\ \frac{-2i\beta}{\alpha^2+\beta^2-2\alpha+1}+2\left(\frac{-2i\beta}{(\alpha^2+\beta^2)-4\alpha+4}\right)+$$

$$3\left(\frac{-2i\beta}{\alpha^2+\beta^2-6\alpha+9}\right)=0$$

$$\Rightarrow\ \frac{\beta}{\alpha^2+\beta^2+1+2\alpha}+\frac{4\beta}{\alpha^2+\beta^2-4\alpha+4}+$$

$$\frac{6\beta}{\alpha^2+\beta^2-6\alpha+9}=0$$

$$\Rightarrow\ \frac{\beta}{(\alpha-1)^2+\beta^2}+\frac{4\beta}{(\alpha-2)^2+\beta^2}+\frac{6\beta}{(\alpha-3)^2+\beta^2}=0$$

It is possible only when $\beta = 0$, but if $\beta = 0$. It means imaginary part of $\alpha \pm i\beta$ is zero so it becomes purely real and this contradicts the assumption, so all of it's roots are real, hence number of real roots of this equation is three.

71. Answer (2)

$f(x) = ax^2 + bx + c$

Let $g(x) = \dfrac{ax^3}{3}+\dfrac{bx^2}{2}+cx$

$\Rightarrow g(0) = 0$

$\Rightarrow g(1) = \dfrac{a}{3}+\dfrac{b}{2}+c=\dfrac{6c+3b+2a}{6}=0$

So, $f(x) = g'(x)$ has at least one root in the interval (0, 1). (By Rolle's Theorem)

72. Answer (1)

Let $x_1 = 2k_1 + 1,\ x_2 = 2k_2 + 1 \ldots. x_5 = 2k_5 + 1$ $(k_1, k_2, k_3, \in I)$

$2(k_1 + k_2 + k_3 + k_4 + k_5) + 5 = 25$

$\Rightarrow k_1 + k_2 + k_3 + k_4 + k_5 = 10$

The minimum value of k_1 is zero, and maximum value of k_1 is 10.

So, total number of non-negative integral solutions will be

$= {}^{10+5-1}C_{5-1} = {}^{14}C_4 = \dfrac{14\times13\times12\times11}{4\times3\times2\times1}$

$= 7 \times 13 \times 11$

$= 77 \times 13$

$= 1001$

73. Answer (3)

Total number of required triangles will be

$= {}^{20}C_3 - 5\cdot{}^4C_3$

$= \dfrac{20\times19\times18}{3\times2\times1} - 5\times4$

= 20 × 19 × 3 – 20

= 20 (57 – 1)

= 20 × 56

= 1120

74. Answer (1)

Following cases arise

(i) 8 balls (1 + 7) = $\dfrac{8!}{1!\cdot7!}\times2 = 16$

(ii) 8 balls (2 + 6) = $\dfrac{8!}{2!\cdot6!}\times2 = 56$

(iii) 8 balls (3 + 5) = $\dfrac{8!}{3!\cdot5!}\times2 = 112$

(iv) 8 balls (4 + 4) = $\dfrac{8!}{4!\cdot4!} = 70$

Total number of ways = 16 + 56 + 112 + 70

= 254 ways

75. Answer (1)

Use transformation $x \to x - 3$

76. Answer (1)

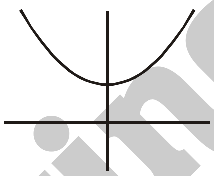
77. Answer (4)

78. Answer (4)

Statement 1 is wrong, statement 2 is correct.

79. Answer (3)

$ax^2 + bx + c = 0$, all of its roots are imaginary if $f(0) = c > 0$ then its graph will be an upward parabola, so $a > 0$



Its value at every point will be greater than 0

So $f(0) > 0$ and $f(1) > 0$ and $f(x) > 0\ \forall\ x \in R$

$\Rightarrow a + b + c > 0$

80. Answer (3)

$\{1 + (x^2 - x)\}^5 = {}^5C_0 + {}^5C_1(x^2 - x) + {}^5C_2(x^2 - x)^2 + {}^5C_3(x^2 - x)^3 + {}^5C_4(x^2 - x)^4 + {}^5C_5(x^2 - x)^5$

$= {}^5C_0 + {}^5C_1(x^2 - x) + {}^5C_2(x^4 + x^2 - 2x^3) + {}^5C_3(x^6 - x^3 - 3x^5 + 3x^4)$........

So the coefficient of x^3 is $= {}^5C_2(-2) - {}^5C_3$

$= -20 - 10$

$= -30$

And the coefficient of x^2 is $= {}^5C_1 + {}^5C_2$

$= 5 + 10$

$= 15 \neq 20$

81. Answer (4)

$${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$$

$= {}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_4 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3$

$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3$

$= {}^{55}C_4 + {}^{55}C_3$

$= {}^{56}C_4$

Statement-1 is wrong, Statement-2 is true.

82. Answer (2)

$|z + i| = |z - 2|$

$|z - (-i)| = |z - (2 + 0i)|$

So z is a locus of a point which is equidistant from $(-i)$ & $(2 + 0i)$ so its locus will be the straight line which is perpendicular bisector of the line joining the points $(-i)$ & $(2 + 0i)$

83. Answer (4)

$|z + 1|^2 + |z - 1|^2 = 1$

Let $z = x + iy$

$|(x + 1) + iy|^2 + |(x - 1) + iy|^2 = 1$

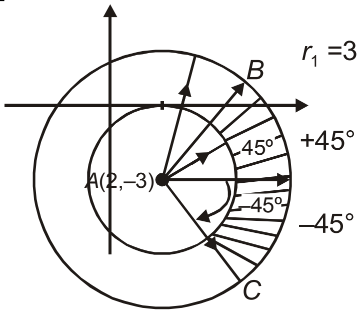
$(x + 1)^2 + y^2 + (x - 1)^2 + y^2 = 1$

$(x^2 + 2x + 1 + y^2) + (x^2 - 2x + 1 + y^2) = 1$

$\Rightarrow 2x^2 + 2y^2 = -1$

or $x^2 + y^2 = -1/z$ (No locus)

84. Answer (3)

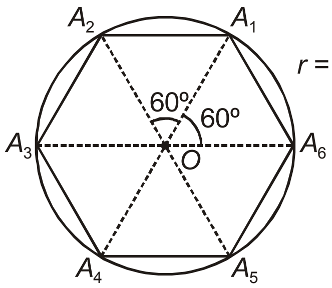


Required area will be $\frac{90^\circ}{360^\circ}\left\{\pi(5)^2-\pi(3)^2\right\}$

$= \frac{1}{4}\times 16\pi$

$= 4\pi$

85. Answer (1)



Area of this resulting figure will be

$= 6 \times (2)^2 \times \frac{\sqrt{3}}{4}$

$= 6\sqrt{3}$ unit^2

86. Answer (2)

$(1 + x)^{10} = a_0 + a_1x + a_2x^2 + a_3x^3.......a_{10}x^{10}$

On putting $x = i$,

$(1 + i)^{10} = a_0 + a_1i + a_2(i)^2 + a_3(i)^3 + a_4(i)^4....a_{10}(i)^{10}$ ...(i)

$\Rightarrow$ $(1 + i^2 + 2i)^5 = (a_0 - a_2 + a_4 - a_6 + a_8 - a_{10}) + i(a_1 - a_3 + a_5 - a_7 + a_9)$

$\Rightarrow$ $32i = (a_0 - a_2 + a_4 - a_6 + a_8 - a_{10}) + i(a_1 - a_3 + a_5 - a_7 + a_9)$

Taking modulus of both sides, we get

$(a_0 - a_2 + ... - a_{10})^2 + (a_1 - a_3 + ... + a_9)^2 = 2^{10}$

87. Answer (1)

$\alpha = \frac{1}{4-3i} = \frac{4+3i}{25}$

$\beta = \bar{\alpha} = \frac{4-3i}{25}$

So, sum of roots = $\frac{8}{25} = \frac{-b}{a}$

Product of roots = $\frac{1}{a} = \frac{1}{25}$

$\Rightarrow a = 25$

and, $b = -8$

88. Answer (1)

$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

$\Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$

$\Rightarrow -\frac{b}{a} = \frac{b^2 - 2ca}{c^2}$

$\Rightarrow -bc^2 = ab^2 - 2ca^2$

or $2ca^2 = ab^2 + bc^2$ ...(i)

$2 = \frac{b^2}{ac} + \frac{bc}{a^2}$

89. Answer (3)

SCHOOL {C,H, L, O, O, S}

Number of words starting from C is = $\frac{5!}{2!} = 60$

Number of words starting from H is = $\frac{5!}{2!} = 60$

Number of words starting from L is = $\frac{5!}{2!} = 60$

Number of words starting from O is = 5! = 120

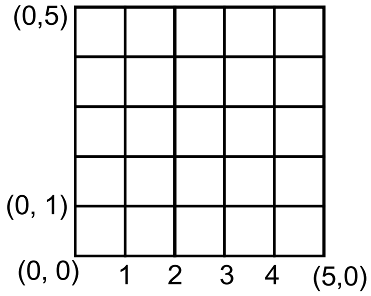
Number of words starting from SCHL = $\frac{2!}{2!} = 1$

Number of words starting from SCHOLO = 1

Number of words starting from SCHOOL = 1

The rank of school will be equal to = 60 + 60 + 60 + 120 + 1 + 1 + 1 = 303

90. Answer (3)



Total number of required triangles is

$= \quad 4\,(1^2 + 2^2 + 3^2 + 4^2 + 5^2)$

$= \quad \frac{4 \times 5 \times 6 \times 11}{6} = 55 \times 4 = 220$.

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