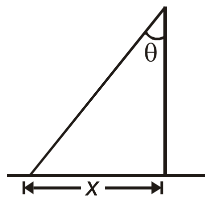
Sankalp IIT

A MUST DO TEST SERIES FOR SURE SHOT SUCCESS IN JEE MAIN AND ADVANCED

Solutions All India Test Series
Test-1

PHYSICS

1. Answer (1)
2. Answer (3)
3. Answer (2)
4. Answer (3)
5. Answer (4)



$x = 3\tan\theta$

$0.6 = 3(\sec^2 45)\ \omega \Rightarrow \omega = 0.1$ rad/sec

6. Answer (2)
7. Answer (3)

Acceleration of A and B both is 9.8 m/s^2 downwards
$\Rightarrow$ relative motion between them will be uniform.

8. Answer (3)
9. Answer (1)

$V_{cart} = 4\hat{i}$

$V_{stone,\ cart} =$

$$6\sin 30^\circ \hat{j} + 6\cos 30^\circ \hat{k} = (3j + 3\sqrt{3}\hat{k})\text{m/s}$$

$\therefore \quad V_{stone} = 4\hat{i} + 3\hat{j} + 3\sqrt{3}\hat{k}$

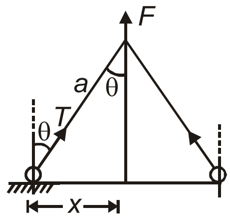
10. Answer (2)
11. Answer (2)

$$T\sin\theta = \frac{mv^2}{R}$$

$T\cos\theta = mg$

$1 = \frac{v^2}{Rg} \Rightarrow v^2 = 10 \times 10 \Rightarrow v = 10$ m/s

12. Answer (3)
13. Answer (1)
14. Answer (4)



$2T\cos\theta = F$

$T\cos\theta + N = mg$

$$N = mg - \frac{F}{2}$$

15. Answer (2)
16. Answer (2)
17. Answer (2)
18. Answer (4)
19. Answer (1)
20. Answer (3)

For one dimensional motion, angle between velocity and acceleration may be zero or 180°

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21. Answer (3)
22. Answer (3)
23. Answer (3)
24. Answer (1)
25. Answer (4)

$\mu = \tan\theta$

$$S = \frac{{v_0}^2}{2a} = \frac{{v_0}^2}{4g\sin\theta}$$

26. Answer (1)

The angle of contact = 5π

$2mg = mge^{\mu.(5\pi)}$

$$\mu = \frac{1}{5\pi}\ln(3)$$

27. Answer (4)

At $t = 0$, particle is at $(a, 0)$

at $t = \frac{\pi}{2P}$, particle is at $(0, b)$

28. Answer (1)

$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

$$t_2 = \sqrt{\frac{2h}{g}}$$

$$\Rightarrow t = t_1 + t_2 = \sqrt{\frac{2}{g}}\left[\sqrt{(H-h)} + \sqrt{h}\right]$$

29. Answer (1)
30. Answer (3)

CHEMISTRY

31. Answer (2)

Due to resonance

32. Answer (3)

Fact.

33. Answer (1)

$$P = \frac{dRT}{M}$$

34. Answer (2)

4 mole atoms are present

35. Answer (3)

$K_2Cr_2O_7 + 14HCl \rightarrow 2KCl + 2CrCl_3 + 3Cl_2 + 7H_2O$

36. Answer (2)

For He, Z > 1

37. Answer (1)

V.P. is independent to volume of container.

38. Answer (3)

$$\frac{1}{\lambda} = R_H z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

39. Answer (4)

$$r \propto \frac{n^2}{z},\ K.E. \propto \frac{z^2}{n^2},\ F \propto \frac{z^3}{n^4}$$

40. Answer (4)

$n - l - 1$.

41. Answer (3)

Only $2n^2$ rule will be followed.

42. Answer (4)

V = KT.

43. Answer (1)

$$V = \frac{2\pi KZe^2}{nh}$$

44. Answer (1)
45. Answer (3)
46. Answer (1)

O_2 is limiting reagent.

47. Answer (4)

In case of weak acid, heat of neutralisation is less than 13.7 kcal.

48. Answer (1)

Fact.

49. Answer (2)

Fact.

50. Answer (2)
51. Answer (1)
52. Answer (1)

three bond moments are cancelled.

53. Answer (3)

Both moment in para form is additive.

54. Answer (2)

Due to its structure.

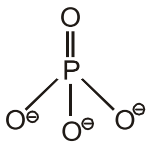
55. Answer (4)

56. Answer (1)

57. Answer (3)

Zinc is not completely consumed.

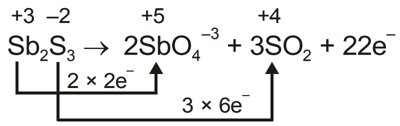
58. Answer (4)



59. Answer (3)

Mn^{+3}, n = 4

60. Answer (2)



MATHEMATICS

61. Answer (1)

We have,

$$3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2-2\cos^2 x}=28$$

$$3^{\sin 2x+2\cos^2 x}+\frac{27}{3^{\sin 2x+2\cos^2 x}}=28$$

Let $3^{\sin 2x+2\cos^2 x}=t$

$$t+\frac{27}{t}=28$$

$\Rightarrow t = 1$ or 27

so $3^{\sin 2x+2\cos^2 x}=1$ or 27

$\Rightarrow \sin 2x + 2\cos^2 x = 0$ or $\sin 2x + 2\cos^2 x = 3$

$\sin 2x + 2\cos^2 x = 3$ is not possible because $\sin 2x$ & $\cos^2 x$ cannot attain their maximum values simultaneously.

$\Rightarrow \sin 2x = -2\cos^2 x$

$\Rightarrow \tan x = -1$

62. Answer (3)

$2\sin\alpha\cos\beta\sin y = \sin\beta\{\sin\alpha\cos y + \cos\alpha\sin y)$

$\Rightarrow 2\sin\alpha\cos\beta\sin y = \sin\alpha\sin\beta\cos y + \cos\alpha\sin\beta\sin y$ …(i)

dividing the equation (i) by $\sin\alpha\sin\beta\sin y$

$2\cot\beta = \cot y + \cot\alpha$

$\Rightarrow \cot\alpha, \cot\beta, \cot y$ are in AP

So, $\tan\alpha$, $\tan\beta$ and $\tan y$ will be in H.P.

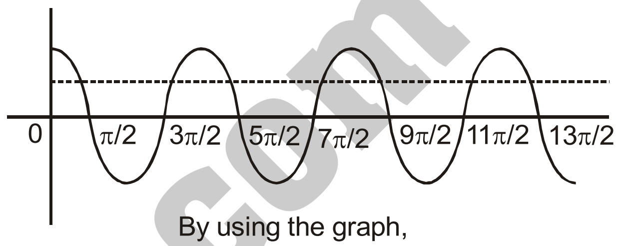
63. Answer (1)

$2\tan^2 x - 5\sec x = 1$

$2(\sec^2 x - 1) - 5\sec x = 1$

$2\sec^2 x - 5\sec x - 3 = 0$

on solving, $\sec x = 3$ or $-\frac{1}{2}$



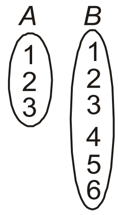
By using the graph,
the least value of n will be 13

$\Rightarrow \cos x = \frac{1}{3}$ or –2

$\cos x = -2$ is not possible

so $\cos x=\frac{1}{3}$ is the only solution.

64. Answer (4)



Case-I → Number of functions satisfying the condition that, if $i > j$ then $f(i) > f(j)$, will be = ${}^6C_3 = 20$

Case-II → Now if $f(1) = f(2) = f(3)$, then number of such functions = 6

Case-III → Now if $f(1) = f(2) < f(3)$, then number of such functions = 5 + 4 + 3 + 2 + 1 = 15

Case-IV → Now if $f(2) = f(3) > f(1)$, then number of such functions will be = 5 + 4 + 3 + 2 + 1 = 15

Case-V → Now if $f(1) = f(3)$ and $f(2)$ is such that $f(1) \neq f(2)$ and $f(2) \neq f(3)$ but satisfy the relation $f(1) < f(2) < f(3)$ such functions are zero so total functions = 20 + 6 + 15 + 15 = 56.

65. Answer (1)

$x^2 - kx + \sin^{-1}(\sin(\pi - 4)) > 0$

$\Rightarrow x^2 - kx + (\pi - 4) > 0$ (This is a quadratic expression)

so, if $a > 0$, the $D < 0$

$\Rightarrow k^2 - 4(\pi - 4) < 0$

or $k^2 < 4(\pi - 4)$

$4(\pi - 4)$ is negative

and k^2 is always positive, hence there is no solution.

66. Answer (4)

$f(x) = \cos^2 x + \sin^2 x$

= 1 (a constant function)

Every constant function is a periodic function having no fundamental period.

67. Answer (1)

We know $\forall\, x \in R - \{0\},\ \frac{x}{\tan^{-1} x} > 1$

So $\frac{x}{\tan^{-1} x} > 1$, similarly $\frac{y}{\tan^{-1} y} > 1$ and $\frac{z}{\tan^{-1} z} > 1$.

$\Rightarrow \frac{x}{\tan^{-1} x} + \frac{y}{\tan^{-1} y} + \frac{z}{\tan^{-1} z} > 3\ \ \forall\, x, y, z \in R$

So there is no solution.

68. Answer (2)

Let $\theta = 20°$, then $3\theta = 60°$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$\sqrt{3} = \frac{3\tan 20° - \tan^3 20°}{1 - 3\tan^2 20°} \quad \{\text{Let } \tan 20° = x\}$$

$$\Rightarrow \sqrt{3} = \frac{3x - x^3}{1 - 3x^2}$$

or $3(1 - 3x^2)^2 = (3x - x^3)^2$

or $3(1 + 9x^4 - 6x^2) = x^6 + 9x^2 - 6x^4$

or $x^6 - 33x^4 + 27x^2 = 3$

$\Rightarrow \tan^6 20° - 33\tan^4 20° + 27\tan^2 20° = 3$

69. Answer (3)

on squaring and adding equation (1) and (2)

$2 + 2\{\cos(a - b)\} = \frac{2}{4} + \frac{6}{4} = 2$

$\Rightarrow \cos(a - b) = 0$ ...(3)

Multiplying (1) and (2)

$\sin a \cos a + \sin b \cos a + \sin a \cos b + \sin b.\cos b = \frac{\sqrt{12}}{4}$

$$\Rightarrow \frac{1}{2}(\sin 2a + \sin 2b) + \sin(a + b) = \frac{\sqrt{12}}{4}$$

$$\frac{1}{2}(2\sin(a + b).\cos(a - b)) + \sin(a + b) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin(a + b) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow a + b = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

70. Answer (3)

$$\sin\frac{2\pi x}{10!} + \cos\frac{\pi x}{9!}$$

L.C.M. of 10! and 2.9! is 10.9! = 10!

71. Answer (2)

$$\frac{\tan(180° + 35°) - \tan(90° + 35°)}{\tan(270° - 35°) + \tan(360° - 35°)}$$

$$= \frac{a + \frac{1}{a}}{\frac{1}{a} - a}$$

$$= \frac{a^2 + 1}{1 - a^2}$$

72. Answer (4)

Given $f(x) = 1 - 2^{-x}$

$\Rightarrow y - 1 = -2^{-x}$

$\Rightarrow (1 - y) = -2^{-x}$

$\Rightarrow -x = \log_2(1 - y)$

$\Rightarrow x = -\log_2(1 - y)$

$\therefore f^{-1}(x) = -\log_2(1 - x)$

73. Answer (3)

$x^2 + 4x + 5 \geq 1$

$\therefore$ Range is $[0, \infty)$

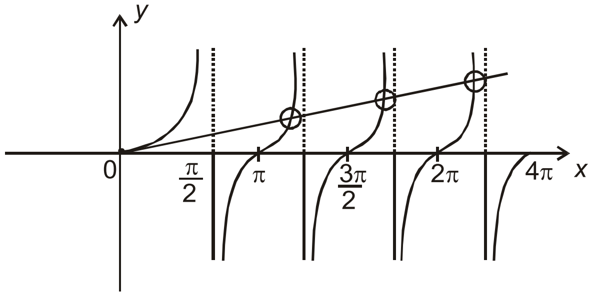
74. Answer (4)

$$\frac{\cos^2\alpha}{\sin^4\alpha + \cos^4\alpha}$$

$$= \frac{\sec^2\alpha}{1 + \tan^4\alpha} = \frac{1 + \tan^2\alpha}{1 + \tan^4\alpha}$$

$$= \frac{10}{82} = \frac{5}{41}$$

75. Answer (2)



76. Answer (3)

We have,

$f'(x) = 3ax^2 + 2x + 2$

$D < 0$

$4 - 24a < 0$

$a > \frac{1}{6}$

77. Answer (1)

$f(x) = 3x + \cos x$

$f'(x) = 3 - \sin x > 0$

So $f(x)$ is an strictly increasing function, and its range is all real numbers.

So function $f(x)$ is one-one as well as onto function so it is a bijective function, hence it is an invertible function.

78. Answer (4)

Let $[\cot^{-1}x] = t$ so that the given intequality reduces to

$t^2 - 6t + 9 \le 0$

$(t - 3)^2 \le 0$

So, this is true only when $t = 3$

$\Rightarrow$ $[\cot^{-1}x] = 3$

$\Rightarrow$ $3 \le \cot^{-1}x < \pi$

$\Rightarrow$ $x \in (-\infty, \cot 3]$

Statement 1 is wrong, statement 2 is correct

79. Answer (3)

80. Answer (1)

81. Answer (4)

Sol. 82 to 84

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = \frac{11}{6}$ ...(i)

$\cos\theta_1 . \cos\theta_2 + \cos\theta_2.\cos\theta_3 + \cos\theta_1.\cos\theta_3 = 1$ ...(ii)

$\cos\theta_1 . \cos\theta_2 . \cos\theta_3 = 1/6$ ...(iii)

from (i)

82. Answer (4)

We have,

$$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = \frac{11}{6}$$

$$2\cos^2\frac{\theta_1}{2} - 1 + 2\cos^2\frac{\theta_2}{2} - 1 + 2\cos^2\frac{\theta_3}{3} - 1 = \frac{11}{6}$$

$$\cos^2\frac{\theta_1}{2} + \cos^2\frac{\theta_2}{2} + \cos^2\frac{\theta_3}{2}$$

$$= \frac{\frac{11}{6} + 3}{2}$$

$$= \frac{29}{12}$$

83. Answer (2)

We have,

$\cos^3\theta_1 + \cos^3\theta_2 + \cos^3\theta_3 - 3\cos\theta_1.\cos\theta_2.\cos\theta_2$

$= (\cos\theta_1 + \cos\theta_2 + \cos\theta_1)$

$(\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 - \cos\theta_1.\cos\theta_2.\cos\theta_3)$

$\Rightarrow \cos^3\theta_1 + \cos^3\theta_2 + \cos^3\theta_3 = \frac{1241}{216}$

84. Answer (4)

$\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3 = 2\cos^2\theta_1 + 2\cos^2\theta_2 + 2\cos^2\theta_3 - 3$

$= 2(\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3) - 3$

$= \frac{-10}{36} = \frac{-5}{18}$

85. Answer (4)

Elements to be added are

(1, 1), (2, 2), (3, 3), (4, 4), (3, 2), (1, 2), (2, 1), (4, 3), (4, 1), (1, 4), (3, 4)

86. Answer (2)

$2f(x) + 3f(1/x) = x^2 - 1$ ...(i)

Replacing x by $1/x$

$3f(x) + 2f(1/x) = \frac{1}{x^2} - 1$ ...(ii)

from (i) and (ii)

$\Rightarrow f(x) = \frac{(2x^2 + 3)(1 - x^2)}{5x^2}$

$\rightarrow$ even but not periodic

87. Answer (1)

We have,

$(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

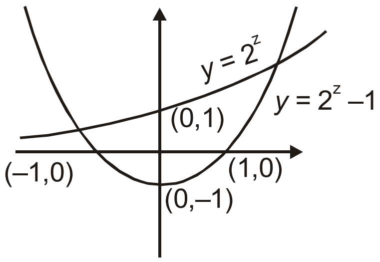
$(1 - \tan^2\theta)(1 + \tan^2\theta) + 2^{\tan^2\theta} = 0$

let, $\tan^2\theta = z$

$(1 - z)(1 + z) + 2^z = 0$

$\Rightarrow$ $2^z = z^2 - 1$

$y = 2^z$, $y = z^2 - 1$ intersect each other at two distinct points

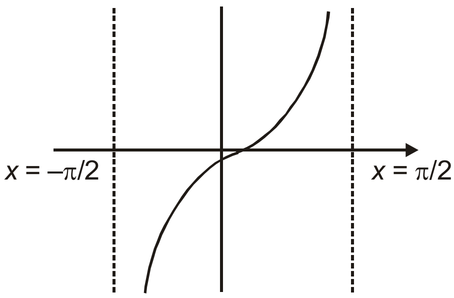


One value of z is negative which is rejected, but the positive value is selected

So, $\tan^2\theta$ = positive

$\Rightarrow$ $\tan\theta$ = One positive value and one negative value

If θ lies in the interval $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



So, there will be excatly two solution.

88. Answer (2)

Let $\sin^{-1}x = A$, $\sin^{-1}y = B$, $\sin^{-1}z = C$

$\Rightarrow$ $A + B + C = \pi$

$\Rightarrow$ $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$

$\Rightarrow$ $\sin A \cos A + \sin B \cos B + \sin C \cos C = 2\sin A \sin B \sin C$

$\Rightarrow$ $x\sqrt{1-x^2} + y\sqrt{1-y^2} + 1\,z\sqrt{1-z^2} = 2xyz$

89. Answer (2)

The range of $12\sin x + 5\cos x$ is $[-13, 13]$ and minimum value of

$2y^2 - 8y + 21$ will be

$= 2(y^2 - 4y + 4) + 13$

$= 2(y - 2)^2 + 13$

So its range will be $[13, \infty)$

So chances of solution are, if L.H.S. & R.H.S. both are equal to 13

So $y = 2$, & $12\sin x + 5\cos x = 13$

$12\sin x + 5\cos x$ will attain its maximum value only one time in $[0, 2\pi]$

So total number of ordered pairs will be 5.

90. Answer (2)

We observe that

$T_1 = \tan^{-1}(1) - \tan^{-1}(0)$

$T_2 = \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{2-1}{1+2\cdot 1}\right) = \tan^{-1}(2) - \tan^{-1}(1)$

$T_3 = \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{3-2}{1+3\cdot 2}\right) = \tan^{-1}(3) - \tan^{-1}(2)$

$T_4 = \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{4-3}{1+4\cdot 3}\right) = \tan^{-1}(4) - \tan^{-1}(3)$

....

....

on adding

$T_1 + T_2 + T_3 ....T_{10} = \tan^{-1}(10) - \tan^{-1}(0)$

$= \tan^{-1}(10)$